Product Acceptance Sampling DESIGN JOURNEY

An Example of Applying Two-Point Statistical Sampling Plans

NOTE: Newer software versions have redesigned input screens

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This Design Journey takes you step-by-step through the design of an acceptance sampling plan. The purpose of the plan will be to guide product acceptance decisions. These decisions will impact customer satisfaction, sales, and profitability. The goal will be to provide meaningful data-driven decisions. We will use software programs TP105 and TP414. v1.2

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PRODUCT ACCEPTANCE REQUIREMENTS

This design journey is about the hypothetical Product XXX. The sampling requirement for Product XXX is as follows:

REJECTABLE QUALITY LEVEL:

The marketing and customer service departments recommend that lots containing 0.10 fraction defective *should not be released* for sale to customers.

ACCEPTABLE QUALITY LEVEL:

The manufacturing plant recommends that lots meeting their current capability of .01 fraction defective *should be released* for sale to customers.

THE CURRENT SAMPLING PLAN:

Our first step will be to analyze the current sampling plan. This plan requires a sample of size n=3 items from a lot of size N=100 items. The lot is accepted if all 3 conform to specification. That is, the acceptance number is c=0 defectives.

EVALUATE THE CURRENT SAMPLING PLAN:

We will use these two questions to evaluate the current sampling plan:

1) What can you tell manufacturing about the producer's alpha risk:
   (that if a lot is .01 fraction defective, it will be rejected?)
2) What can you tell marketing about the consumers' beta risk:
   (that if a lot is .10 fraction defective, it will be accepted?)

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design      Sampling Plans for ATTRIBUTES

          PRODUCT OR PROCESS NAME:  PRODUCT XXX
 ATTRIBUTE OR COUNT DATA  (A or C)  :  A
  NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
         NAME OF THE SAMPLED UNIT:  UNIT


        KIND OF INPUT  (P, D, or S):  D     <---

 SEQUENTIAL OR FIXED-N PLAN (S or F):  F
                     ENTER N, C = . 3 0   <---

D-RULE, P-CURVES, S-STATS(D, P, or S):  D     <---


OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F     APPEND/WRITEOVER (A/W)? W

 AQL=0.016952     RQL=0.631597        ACTUAL ALPHA=.050000   ACTUAL BETA=.050000
```

OUTPUT SCREEN

```
          *****   DESIGN JOURNEY:   PLAN #1    *****

   FIXED SAMPLE SIZE ATTRIBUTE SAMPLING PLAN TO CONTROL
        PRODUCT XXX FRACTION DEFECTIVE UNITS

     PRODUCERS  POINT                    CONSUMERS  POINT
   AQL(fractn)= 0.016952               RQL(fractn)= 0.631597
   DESIGN ALPHA= .05                    DESIGN BETA= .05
   ACTUAL ALPHA= .050                   ACTUAL BETA= .050


          DECISION RULE:

          SAMPLE SIZE, n = 3 UNITS
                      AC = 0 DEFECTIVES
                      RE = 1 DEFECTIVES

   BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-26-94
```

STEP 1.1

PURPOSE: Analyze existing sampling plan. n=3, C=0

This input screen shows how an existing sampling plan (n=3, c=0) is input for analysis. Underlining indicates user input.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) The existing sampling plan is a fixed-n attribute plan. Run program: TP105: Two-Point Sampling Plans for Defects and Defectives.
2) After entering n=3 & c=0, you are shown the producer's and consumers' points. See AQL=.017 and RQL=.63 at the bottom of the screen.
3) The program used alpha=.05 and beta=.05 to calculate AQL and RQL.

PROGRAM OPERATION

1) The operating conventions are the same for TP105 for attributes and TP414 for variables.
2) Default values are invoked at all input points by entering <return>. At the time of entry, the default value is shown on screen at the lower right corner.
3) Use the <Up-Arrow> and <Down-Arrow> keys to repeat the previous input entered for that question. If previous input has not been entered, the program's default will be used.

RESULT: Producer's and Consumers' points for n=3, C=0.

This is the "decision rule" output report for the fixed-n sampling plan

INTERPRETATION OF THE OUTPUT REPORT

1) The producer's point shows that if a lot contains AQL=.016952 fraction defective, then the producer's risk of rejecting it is alpha=0.05 (5%).
2) The consumers' point shows that if a lot contains RQL=.631597 fraction defective, then the consumers' risk of accepting it is beta=0.05 (5%).
3) The symbols AC and RE are typically used to represent ACceptance and REjection numbers.

STRATEGY FOR USE

1) Use the producer's and consumers' points to predict if the plan will perform as desired, if implemented.
2) The RQL=.63 fraction defective is obviously too high.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, we will answer the two questions proposed on page 1 by analyzing the OC-Curve for n=3, C=0.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design      Sampling Plans for ATTRIBUTES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
  ATTRIBUTE OR COUNT DATA  (A or C)  :  A
   NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
            NAME OF THE SAMPLED UNIT:  UNIT


          KIND OF INPUT  (P, D, or S):  D

  SEQUENTIAL OR FIXED-N PLAN (S or F):  F
                        ENTER N, C =  3 0

D-RULE, P-CURVES, S-STATS(D, P, or S):  P    <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  G    <---


            SHADE GRAPH LINE (N/Y):  N
OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F    APPEND/WRITEOVER (A/W)? A
   GRAPH X-AXIS (FRACTN): XMIN,XMAX =  0 .3 <---

 AQL=0.016952    RQL=0.631597       ACTUAL ALPHA=.050000    ACTUAL BETA=.050000
```

STEP 1.2

PURPOSE: Analyze existing sampling plan n=3, C=0 with OC-Curve.

This input screen shows how an OC-Curve is produced. This OC-Curve will show how well the plan n=3, C=0 discriminates between quality levels.

SAMPLING PLAN DESIGN CONSIDERATIONS

An OC-Curve will analyze the performance of the sampling plan n=3, C=0 by plotting Pa vs p':

Pa = Probability of Acceptance.

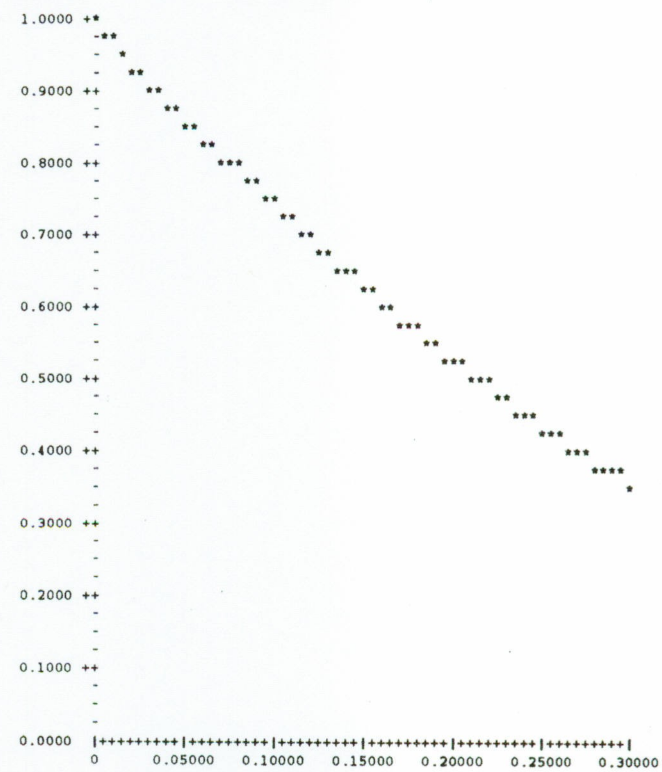
p' = the true lot fraction defective.

PROGRAM OPERATION

1) The OC-Curve for n=3, C=0 is selected by entering P for performance curves on the line: (D, P, or S):.
2) An "EXPLANATION LINE" at the bottom of the input screen clarifies this choice, but it only appears when the cursor is on the prompt to be explained.
3) The OC-Curve will be in graphic form by entering G on the next line.
4) The horizontal axis limit XMAX was set to .3 fraction defective, taking advantage of the six major scale intervals to produce exact rounded axis labels.

OUTPUT SCREEN

```
                                   *****  DESIGN JOURNEY:  PLAN #1   *****
PROBABILITY
    OF                        OPERATING CHARACTERISTIC CURVE TO CONTROL
ACCEPTANCE                      PRODUCT XXX FRACTION DEFECTIVE UNITS
```




```
CRITERIA FOR SAMPLING PRODUCT XXX UNITS:

         PRODUCERS  POINT                     CONSUMERS  POINT

        AQL(fractn)= 0.016952               RQL(fractn)= 0.631597
        DESIGN ALPHA= .05                   DESIGN BETA= .05
        ACTUAL ALPHA= .050                  ACTUAL BETA= .050

 BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-26-1994
```

THE RESULT: OC-Curve for the plan: n=3, C=0

INTERPRETATION OF THE OUTPUT REPORT

The OC-Curve for n=3, C=0 shows that a lot having 0.10 fraction defective has Pa=70%. This is not satisfactory, considering marketing's acceptance requirement, page 1, that a 0.10 lot be rejected.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, in step 2, we will improve consumer protection by designing a new plan (n & C) having Pa=.05 for an RQL of 0.10 fraction defective.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design    Sampling Plans for ATTRIBUTES

           PRODUCT OR PROCESS NAME:  PRODUCT XXX
 ATTRIBUTE OR COUNT DATA  (A or C)  :  A
  NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
           NAME OF THE SAMPLED UNIT:  UNIT


       KIND OF INPUT  (P, D, or S):  P   <---

 SEQUENTIAL OR FIXED-N PLAN (S or F):  F
       ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10    <---

D-RULE, P-CURVES, S-STATS(D, P, or S):  D   <---


OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F     APPEND/WRITEOVER (A/W)? A

     n = 43        c = 1        ACTUAL ALPHA=.068947   ACTUAL BETA=.062257
```

STEP 2.1

PURPOSE: Adjust performance to Pa=.05, RQL=.10

This input screen shows how the desired performance, with RQL=0.10, is entered.

SAMPLING PLAN DESIGN CONSIDERATIONS

The alpha and beta risks were chosen to be 0.05 each, which is a fairly typical practice.

From page 1, AQL=0.01 is manufacturing's product lot acceptance requirement based on past process capability. RQL=0.10 is marketing's requirement for rejection.

PROGRAM OPERATION

The calculated decision rule, n=43, C=1, is displayed on the bottom of the screen as soon as the performance, (.05 .05 .01 .10), is input.

USAGE NOTES

The words PRODUCT XXX, DEFECTIVE, UNIT and the TITLE at the top of the output report were user-inputs.

OUTPUT SCREEN

```
          *****   DESIGN JOURNEY:  PLAN #2   *****


   FIXED SAMPLE SIZE ATTRIBUTE SAMPLING PLAN TO CONTROL
        PRODUCT XXX FRACTION DEFECTIVE UNITS

   PRODUCERS  POINT                    CONSUMERS  POINT
 AQL(fractn)= .01                   RQL(fractn)= .10
 DESIGN ALPHA= .05                  DESIGN BETA= .05
 ACTUAL ALPHA= .069                 ACTUAL BETA= .062



      DECISION RULE:

      SAMPLE SIZE, n = 43 UNITS
                 AC = 1 DEFECTIVES
                 RE = 2 DEFECTIVES

  BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-26-1994
```

THE RESULT: The decision rule is: n=43, C=1 for RQL=0.10

DESCRIPTION OF THE OUTPUT REPORT

The report contains the producer's and consumers' points followed by the calculated fixed-n decision rule of n=43, C=1.

INTERPRETATION OF THE OUTPUT REPORT

Note that the actual alpha and beta differ from the design values. This is due to rounding of n and C to whole numbers.

THE NEXT STEP OF THE DESIGN JOURNEY

We will next evaluate the OC-Curve for this plan.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)     Two-Point Method of Design     Sampling Plans for ATTRIBUTES

              PRODUCT OR PROCESS NAME:  PRODUCT XXX
   ATTRIBUTE OR COUNT DATA  (A or C)  :  A
    NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
            NAME OF THE SAMPLED UNIT:  UNIT


           KIND OF INPUT  (P, D, or S):  P

   SEQUENTIAL OR FIXED-N PLAN (S or F):  F
        ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10

D-RULE, P-CURVES, S-STATS(D, P, or S):  P    <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  G    <---


              SHADE GRAPH LINE (N/Y):  Y
OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F     APPEND/WRITEOVER (A/W)? A
    GRAPH X-AXIS (FRACTN): XMIN,XMAX =  0 .3 <---

      n = 43          c = 1            ACTUAL ALPHA=.068947    ACTUAL BETA=.062257
```

STEP 2.2

PURPOSE: Analyze performance with of n=43, C=1 with an OC-Curve graph.

This input screen shows how an OC-Curve is produced by inputting the producers and consumer's points.

SAMPLING PLAN DESIGN CONSIDERATIONS

We will use the OC-Curve will evaluate the plan n=43, C=1 and compare it to the existing C=0 plan of step 1.2.

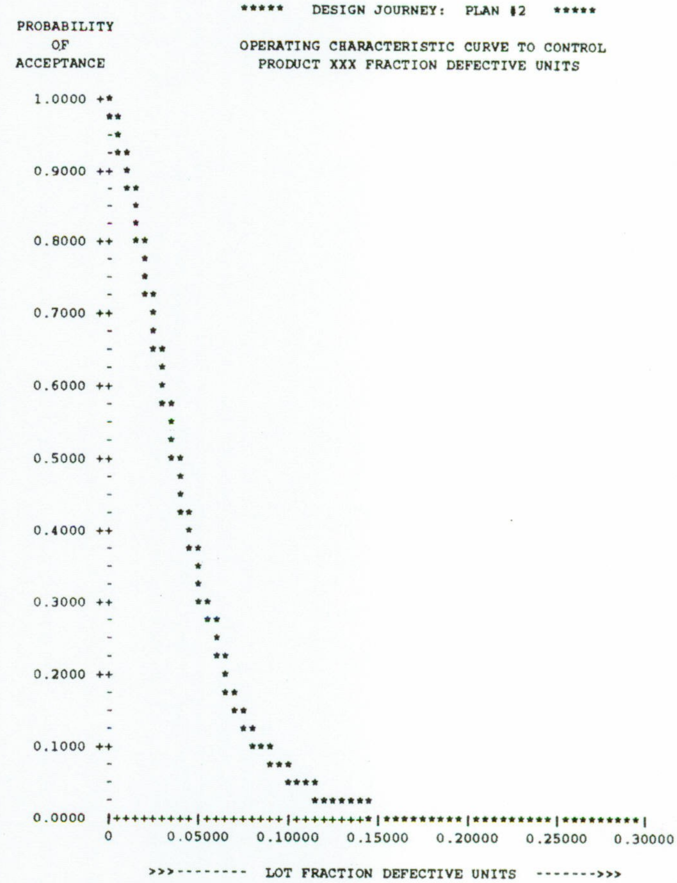
PROGRAM OPERATION

We entered XMAX=.3 so that the OC-Curve will have the same scale as the n=3, C=0 plan of step 1.2.

OUTPUT SCREEN

```
***** DESIGN JOURNEY: PLAN #2 *****

OPERATING CHARACTERISTIC CURVE TO CONTROL
PRODUCT XXX FRACTION DEFECTIVE UNITS
```




```
CRITERIA FOR SAMPLING PRODUCT XXX UNITS:

      PRODUCERS  POINT                    CONSUMERS  POINT

      AQL(fractn)= .01                    RQL(fractn)= .10
      DESIGN ALPHA= .05                   DESIGN BETA= .05

BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-26-1994
```

THE RESULT: OC-Curve for n=43, C=1

INTERPRETATION OF THE OUTPUT REPORT

1) This OC-Curve shows that a lot at AQL=0.01 will have high probability of acceptance, while a lot at RQL=0.10 have low probability of acceptance..
2) This plan will inspect 43% of the N=100 lot size. It meets the required RQL=.10, but at the expense of too much inspection.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, in step 3, we will reduce the amount of inspection with a sequential sampling plan.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design     Sampling Plans for ATTRIBUTES

              PRODUCT OR PROCESS NAME:  PRODUCT XXX
  ATTRIBUTE OR COUNT DATA  (A or C)  :  A
   NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
             NAME OF THE SAMPLED UNIT:  UNIT


         KIND OF INPUT  (P, D, or S):  P

  SEQUENTIAL OR FIXED-N PLAN (S or F):  S   <---
        ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10

D-RULE, P-CURVES, S-STATS(D, P, or S):  D    <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  G    <---

              TRUNCATION SAMPLE SIZE =  65   <---

OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F    APPEND/WRITEOVER (A/W)? A
GRAPH AXES MAXIMUMS:  Y(# def), X(n) =  5 66 <---

      n = 43        c = 1          ACTUAL ALPHA=.068947   ACTUAL BETA=.062257
```

STEP 3.1

PURPOSE: Design sequential plan graph to match n=43, C=1.

This input screen shows how to design a sequential sampling plan graph.

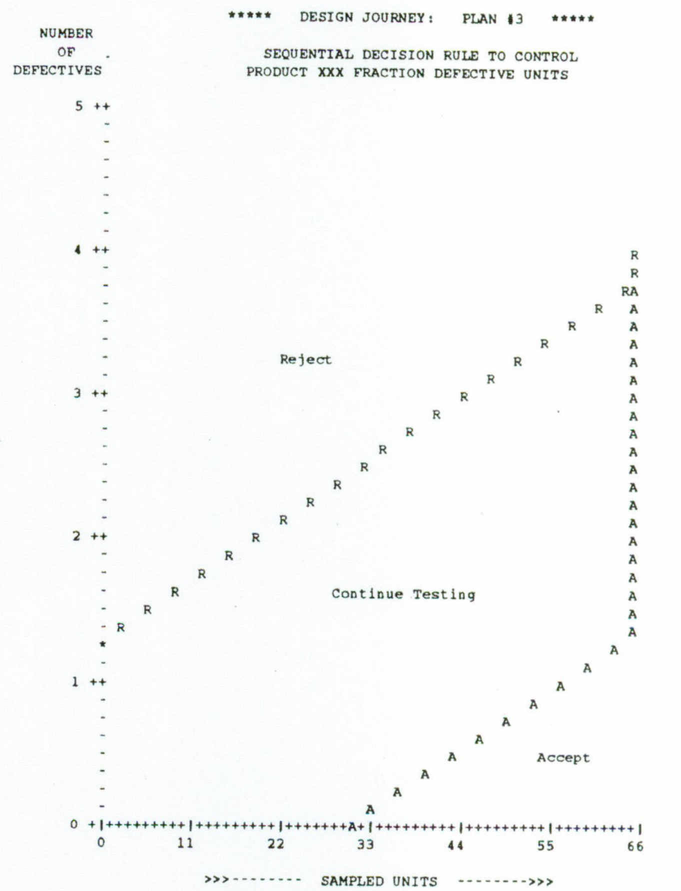
SAMPLING PLAN DESIGN CONSIDERATIONS

1) A sequential sampling plan builds up the sample one or more items at a time. The decision to stop sampling and accept or reject is made by comparing the number of defects at sample size n to the acceptance line and rejection line.
2) A sequential sampling plan usually requires less sampling than a fixed-n plan having the same OC-Curve. An exception to this is the C=0 plan, for which sequential sampling cannot reduce the sample size.
3) The truncation sample size should be set to a value that will rarely be reached. Several rules of thumb are used, but we choose the default of 1.5*fixed-n = 1.5*43 = 65

PROGRAM OPERATION

At the last prompt shown -- graph axis maximums -- press <enter> for automatic scaling.

OUTPUT SCREEN




```
CRITERIA FOR SAMPLING PRODUCT XXX FOR DEFECTIVES:

PRODUCERS  POINT      CONSUMERS  POINT     MINIMUM AND MAXIMUM SAMPLE SIZES (N)

AQL(fractn)= .01      RQL(fractn)= .10     MIN N/ACC =31     MAXIMUM N =65
DESIGN ALPHA= .05     DESIGN BETA= .05     MIN N/REJ =2      'MAX' ASN =40

BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-24-1994
```

THE RESULT: Graph of the sequential decision rule.

DESCRIPTION OF THE OUTPUT REPORT

Note the three zones: Acceptance, Continue, Rejection

INTERPRETATION OF THE REPORT:

The final lines of the report show important characteristics of the graph:

1) the minimum n to accept is 31, with 0 defectives.
2) the minimum n to reject is 2, with 2 defectives.
3) the truncation n is 65, user specified.
4) and the maximum of the ASN-Curve is 40.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, we will produce a sequential *decision table*.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design    Sampling Plans for ATTRIBUTES

           PRODUCT OR PROCESS NAME:  PRODUCT XXX
   ATTRIBUTE OR COUNT DATA  (A or C):  A
  NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
           NAME OF THE SAMPLED UNIT:  UNIT


        KIND OF INPUT  (P, D, or S):  P

SEQUENTIAL OR FIXED-N PLAN (S or F):  S
      ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10

D-RULE, P-CURVES, S-STATS(D, P, or S):  D   <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  T   <---
           KIND OF TABLE (N or X):  N
          TRUNCATION SAMPLE SIZE =  65  <---

OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F    APPEND/WRITEOVER (A/W)?

n =43 UNITS,  c =1 DEFECTIVES      ACTUAL ALPHA=.068947  ACTUAL BETA=.062257
```

STEP 3.2

PURPOSE: Design a sequential sampling plan table to match n=43, C=1.

This input screen shows how to design a sequential sampling plan table.

SAMPLING PLAN OPERATION

1) Sequential sampling plans are easier to operate with a table than a graph. The sampled items are accumulated sequentially. The number of defectives is compared to the Ac and Re numbers as the sample is accumulated.
2) The table shows that for this plan, decisions cannot be made on the first item. Item #2 through #30 can cause rejection, but not acceptance. The first acceptance possible is on 31 items with 0 defective.
3) If a decision to accept is not made at n=31 (because # defectives = 1 or 2), the next possible acceptance is at item #57.
4) Grouping: Because of (2) and (3), you might decide to collect or examine the items in 3 groups, based on possible acceptances:
   - group 1: 31 units
   - group 2: 26 units, cumulative=31+26=57
   - group 3: 9 units, cumulative=57+ 9=65
6) Note that the table forces truncation at n=65, because Re=Ac+1.
7) Samples will seldom reach truncation at n=65. The ASN-Curve (step 3.3) explains why.

OUTPUT SCREEN

```
*****   DESIGN JOURNEY:   PLAN #3   *****

SEQUENTIAL ATTRIBUTE SAMPLING PLAN TO CONTROL
   PRODUCT XXX FRACTION DEFECTIVE UNITS

  PRODUCERS  POINT                CONSUMERS  POINT
AQL(fractn)= .01                RQL(fractn)= .10
DESIGN ALPHA= .05               DESIGN BETA= .05

SEQUENTIAL DECISION RULE:
COMPARE THE NUMBER OF DEFECTIVES TO AC AND RE:
```

(N) SAMPLE SIZE GROUPING OF UNITS FROM	TO	ACCEPT IF DEFECTIVES ARE EQUAL TO OR LESS THAN: (AC)	REJECT IF DEFECTIVES ARE EQUAL TO OR MORE THAN: (RE)
1	1	*	**
2	19	*	2
20	30	*	3
31	44	0	3
45	56	0	4
57	63	1	4
64	64	2	4
65	65	3	4

```
NOTES:       * = CANNOT ACCEPT
            ** = CANNOT REJECT

BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-22-1994
```

THE RESULT: Decision table for the attribute sequential plan.

DESCRIPTION OF THE OUTPUT REPORT

The Ac and Re columns are points on the acceptance and rejection lines of the graph in step 3.1. They agree with the values at the bottom of the graph. The minimum n to accept is 31, the minimum n to reject is 2 the truncation n is 65.

INTERPRETATION OF THE OUTPUT REPORT

The Ac and Re columns are boundaries of the three areas of the graph. The table may be easier to operate than the graph on the shop floor.

THE NEXT STEP OF THE DESIGN JOURNEY

Next we will evaluate OC, AOQ, ASN, and ARL curves of this plan.Our goal is to describe how the plan will perform in discriminating between various quality levels(OC-Curve), the impact on outgoing quality(AOQ-Curve), and the amount of inspection required to accomplish this(ASN-Curve).

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP105 (v1.21)    Two-Point Method of Design      Sampling Plans for ATTRIBUTES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
   ATTRIBUTE OR COUNT DATA  (A or C)  :  A
    NAME OF THE QUALITY CHARACTERISTIC:  DEFECTIVE
            NAME OF THE SAMPLED UNIT:  UNIT


            KIND OF INPUT  (P, D, or S):  P

   SEQUENTIAL OR FIXED-N PLAN (S or F):  S    <---
         ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10

D-RULE, P-CURVES, S-STATS(D, P, or S):  P   <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  T   <---
         CURVES IN TABLE: (O, N, Q, L):  ALL <---
          STEPPING VARIABLE (O or P):  P   <---

OUTPUT TO SCREEN,PRINTER,FILE (S/P/F):  F     APPEND/WRITEOVER (A/W)? A
FRAC DEFECTIVE SCALE LIMITS: MIN,MAX =  0 .2

      n = 43          c = 1          ACTUAL ALPHA=.068947    ACTUAL BETA=.062257
```

STEP 3.3

PURPOSE: Analyze performance curve table for attribute sequential plan.

This input screen shows how to produce all possible performance curves for the sequential plan that matches the fixed-n plan: n=43, C=1.

SAMPLING PLAN DESIGN CONSIDERATIONS

These performance curves allow you to evaluate the cost and assurance provided by the plan n=43, C=1.

The performance curves are:

Operating Characteristic:	Pa vs p'
Average Outgoing Quality:	AOQ vs p'
Average Sample Number:	ASN vs p'
Average Run Length:	ARL vs p'

A *matched pair* of plans - fixed-n and sequential - share the same the same OC, AOQ, and ARL curves. Only the ASN differs. The fixed-n plan has a constant ASN equal to the fixed sample size. A matched variables sequential plan will have smaller ASNs than the attribute plan.

PROGRAM OPERATION

1) Use the keyword ALL to avoid naming all the individual performance curves.
2) The stepping variable of the table is the column that steps in equal increments. It can be Pa or p'.
3) The stepping variable always appears in the left-hand column of the performance table.

OUTPUT SCREEN

***** DESIGN JOURNEY: PLAN #3 *****

TABLE OF PERFORMANCE CHARACTERISTIC CURVES
OF ATTRIBUTE SAMPLING PLAN TO CONTROL
PRODUCT XXX FRACTION DEFECTIVE UNITS

PRODUCERS POINT	CONSUMERS POINT
AQL(fractn)= .01	RQL(fractn)= .10
DESIGN ALPHA= .05	DESIGN BETA= .05

	TRUE FRACTION DEFECTIVE (p')	PROBABILITY OF ACCEPTANCE (Pa)	AVERAGE OUTGOING QUALITY (AOQ)	AVERAGE SAMPLE NUMBER (ASN)	AVERAGE RUN LENGTH (ARL)
	0.000000	1.000	0.000000	30.89	
	0.005000	0.98484	0.004924	34.27	65.94
(AQL)=	0.010000	0.95000	0.009500	37.15	20.00
	0.01500	0.8974	0.01346	39.44	9.747
	0.02000	0.8295	0.01659	40.98	5.866
	0.02500	0.7504	0.01876	41.70 *	4.007
	0.03000	0.6652	0.01995	41.61	2.987
	0.03500	0.5789	0.02026 *	40.80	2.375
	0.04000	0.4959	0.01984	39.43	1.984
	0.04500	0.4195	0.01888	37.64	1.723
	0.05000	0.3513	0.01756	35.62	1.542
	0.05500	0.2920	0.01606	33.49	1.412
	0.06000	0.2414	0.01448	31.36	1.318
	0.06500	0.1989	0.01293	29.29	1.248
	0.07000	0.1634	0.01144	27.32	1.195
	0.07500	0.1342	0.01006	25.49	1.155
	0.08000	0.1101	0.008806	23.79	1.124
	0.08500	0.09030	0.007676	22.23	1.099
	0.09000	0.07411	0.006670	20.81	1.080
	0.09500	0.06085	0.005781	19.52	1.065
(RQL)=	0.1000	0.05000	0.005000	18.34	1.053
	0.1050	0.04111	0.004317	17.27	1.043
	0.1100	0.03383	0.003721	16.30	1.035
	0.1150	0.02785	0.003203	15.41	1.029
	0.1200	0.02295	0.002754	14.60	1.023
	0.1250	0.01892	0.002365	13.86	1.019
	0.1300	0.01560	0.002028	13.18	1.016
	0.1350	0.01287	0.001737	12.56	1.013
	0.1400	0.01062	0.001487	11.99	1.011
	0.1450	0.008764	0.001271	11.46	1.009
	0.1500	0.007233	0.001085	10.98	1.007
	0.1550	0.005969	0.000925	10.53	1.006
	0.1600	0.004925	0.000788	10.11	1.005
	0.1650	0.004063	0.000670	9.724	1.004
	0.1700	0.003351	0.000570	9.364	1.003
	0.1750	0.002762	0.000483	9.029	1.003
	0.1800	0.002276	0.000410	8.715	1.002
	0.1850	0.001874	0.000347	8.422	1.002
	0.1900	0.001543	0.000293	8.147	1.002
	0.1950	0.001269	0.000247	7.889	1.001
	0.2000	0.001043	0.000209	7.646	1.001

BY SAMPLING PLAN PROGRAM TP105 V1.21 ON 03-26-1994

THE RESULT: OC, AOQ, ASN, and ARL Curves.

DESCRIPTION OF THE OUTPUT REPORT

The AQL and RQL points are marked in the left margin of the performance table. Note that the plan has the following performance when applied to an RQL(p'=.10) lot: Pa=.05, AOQ=.005, ASN=18.34, ARL=1.053.

Note that AOQ and ASN curves have maximums.

THE NEXT STEP OF THE DESIGN JOURNEY

In step 4, we will design a variables sampling plan to further reduce the sample size and cost. It is assumed that the quality characteristic can be measured.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)     Two-Point Method of Design      Sampling Plans for VARIABLES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
                      VARIABLE NAME:  MEASUREM'T
       INPUT FORM, ISL or MEAN (I or M):  I    <---
          NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
    SEQUENTIAL or FIXED-N PLAN (S or F):  F    <---

LOWER, UPPER, OR BOTH     (L, U, or B):  L    <---
                       LOWER SPEC LIMIT =  0    <---
   IS SIGMA KNOWN (Y/N):  N    <---
                 KIND OF INPUT  (P or D):  P
        ENTER ALPHA, BETA, AQL, RQL =  .05  .05  .01  .10
D-RULE, P-CURVES, S-STATS(D, P, or S):  D    <---


OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F     APPEND/WRITEOVER (A/W): A


                  DECIMAL PLACES = _

       n = 27          K = 1.803949
```

OUTPUT SCREEN

```
         *****   DESIGN JOURNEY:  PLAN #4   *****

   FIXED SAMPLE SIZE VARIABLES SAMPLING PLAN TO CONTROL
           FRACTION CONFORMING TO:  LISL = 0
 FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT    SIGMA UNKNOWN

  PRODUCERS POINT                          CONSUMERS POINT
      AQL = .01                                 RQL = .10
    ALPHA = .05                                BETA = .05


    DECISION RULE FOR A FIXED-N SAMPLING PLAN:

    SAMPLE SIZE = 27
    REJECT IF (XBAR-LISL)/SIGMA IS LESS THAN 1.80

  BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994
```

STEP 4.1

PURPOSE: Design a variables fixed-n plan to match the attribute plan.

This input screen of TP414 shows how to design a fixed-n variables sampling plan for unknown-sigma.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) A fixed-n variables sampling plan takes a smaller size sample than a fixed-n attribute plan to match the same OC-Curve.
2) This matched variables plan will have the same producer's and consumers' points.
3) We choose the ISL input form. This provides the performance measures in fraction defective, the same as the attribute plan.
4) The variables ISL plans assume the population is normally distributed. Large departures from normality can often be corrected by applying an normalizing transformation to the data.
5) This is an example of an "Unknown-Sigma" plan. Unknown simply means that the standard deviation is not known from historical data, so it will be calculated from the sample.

PROGRAM OPERATION

Use program TP414: Two-Point Sampling Plans for Variables.

THE RESULT: The variables sampling plan: n=27, k=1.80, unknown-sigma

DESCRIPTION OF THE OUTPUT REPORT

The report contains the producer's and consumers' points followed by the calculated fixed-n decision rule n=27, k=1.80.

INTERPRETATION OF THE OUTPUT REPORT

Interpret k as the distance from the sample average to the specification limit. It is in standard deviation units.

THE NEXT STEP OF THE DESIGN JOURNEY

We will next evaluate the OC-Curve for this variables plan.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)     Two-Point Method of Design     Sampling Plans for VARIABLES

              PRODUCT OR PROCESS NAME:  PRODUCT XXX
                        VARIABLE NAME:  MEASUREM'T
       INPUT FORM, ISL or MEAN (I or M):  I
        NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
     SEQUENTIAL or FIXED-N PLAN (S or F):  F

LOWER, UPPER, OR BOTH      (L, U, or B):  L
                     LOWER SPEC LIMIT =  0
   IS SIGMA KNOWN (Y/N):  N
              KIND OF INPUT  (P or D):  P
         ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10
D-RULE, P-CURVES, S-STATS(D, P, or S):  P    <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  G    <---

OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F    APPEND/WRITEOVER (A/W): A

X-AXIS SCALE LIMITS (FRAC): MIN, MAX =  0 .3 <---

      n = 27          K = 1.803949
```

STEP 4.2

PURPOSE: Analyze the variables OC-Curve.

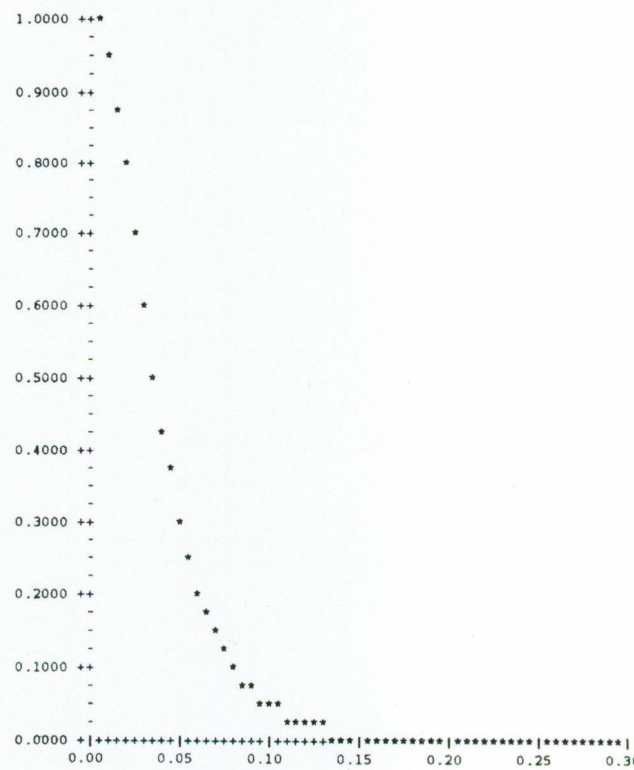
This input screen shows how to make an OC-Curve graph to analyze the variables plan n=27, k=1.80.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) This variables OC-Curve will match that of the previous attribute plan because the same producer's and consumers' points are entered.
2) The horizontal axis of the OC-Curve is be scaled the same as the previous attribute curve to facilitate comparison. See step 2.2.

OUTPUT SCREEN

```
PROBABILITY            ***** DESIGN JOURNEY:  PLAN #4   *****
    OF             OC-CURVE FOR VARIABLES PLAN TO !IS DEFECTIVE
ACCEPTANCE            BASED ON A LOWER SPECIFICATION LIMIT FOR MEASUREM'T
```




```
         ------  FRACTION MEASUREM'T BEYOND LIMIT  ------>

FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT    LISL = 0     SIGMA UNKNOWN

        PRODUCERS POINT                     CONSUMERS POINT
          ALPHA = .05                          BETA = .05
        AQL(FrDf) = .01                      RQL(FrDf) = .10

BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994
```

THE RESULT: Graph of OC-Curve for: n=27, k=1.80.

INTERPRETATION OF THE OUTPUT REPORT

The OC-Curve for variables is almost identical to the attribute curves. See step 2.2. The sample size of 27 is better than 43, but can be improved further.

THE NEXT STEP OF THE DESIGN JOURNEY

Next we will design a *sequential* plan for variables to lower the sample size even more.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)    Two-Point Method of Design      Sampling Plans for VARIABLES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
                      VARIABLE NAME:  MEASUREM'T
       INPUT FORM, ISL or MEAN (I or M):  I   <---
        NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
     SEQUENTIAL or FIXED-N PLAN (S or F):  S   <---
CLASSICAL SEQUENTIAL or TSS  (C or T):  T   <---
LOWER, UPPER, OR BOTH     (L, U, or B):  L
                      LOWER SPEC LIMIT =  0
   IS SIGMA KNOWN (Y/N):  N
            KIND OF INPUT  (P or D):  P
       ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10
D-RULE, P-CURVES, S-STATS(D, P, or S):  D   <---


OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F     APPEND/WRITEOVER (A/W): A


       n = 27          K = 1.803949
```

STEP 5.1

PURPOSE: Design a matching sequential plan, unknown-sigma.

This input screen shows how to design a sequential sampling plan for variables, unknown sigma.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) A sequential plan will reduce the amount of inspection required to make acceptance/rejection decisions.
2) For sequential unknown-sigma plans, use the TSS - not SPR - type. See arrows on the input screen above.
3) It is not possible to design sigma unknown sampling plans based on the mean.

OUTPUT SCREEN

```
        *****   DESIGN JOURNEY:  PLAN #5   *****

   TSS SEQUENTIAL VARIABLES SAMPLING PLAN TO CONTROL
         FRACTION CONFORMING TO:  LISL = 0
FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT   SIGMA UNKNOWN

PRODUCERS POINT                        CONSUMERS POINT
    AQL = .01                                RQL = .10
  ALPHA = .05                               BETA = .05

  DECISION RULE:
       COMPARE DECISION LIMITS (Ac AND Re) TO
                KL= (XBAR - LISL)/SIGMA
  CONTINUE SAMPLING UNTIL ACCEPTANCE OR REJECTION:
```

SAMPLE SIZE (N)	REJECT IF KL IS LESS THAN (Re)	ACCEPT IF KL IS MORE THAN (Ac)
2	-34.69	
3	-3.83	
4	-1.03	
5	-0.20	
6	0.21	27.57
7	0.46	13.32
8	0.63	8.94
9	0.76	6.86
10	0.87	5.66
11	0.96	4.88
12	1.03	4.33
13	1.10	3.92
14	1.16	3.61
15	1.21	3.36
16	1.26	3.15
17	1.30	2.98
18	1.35	2.83
19	1.39	2.69
20	1.43	2.58
21	1.47	2.47
22	1.51	2.37
23	1.55	2.27
24	1.59	2.18
25	1.64	2.08
26	1.70	1.96
27	1.80	1.80

BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994

THE RESULT: Sequential decision rule, unknown-sigma.

DESCRIPTION OF THE OUTPUT REPORT

The report contains the producer's and consumers' points, the decision rule instructions, and the acceptance and rejection columns.

INTERPRETATION OF THE OUTPUT REPORT

Generally, a decision will be made before reaching n=27.

THE NEXT STEP OF THE DESIGN JOURNEY

In step 6 we will further reduce the sample size by designing a plan based on known sigma.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)     Two-Point Method of Design      Sampling Plans for VARIABLES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
                      VARIABLE NAME:  MEASUREM'T
      INPUT FORM, ISL or MEAN (I or M):  I
       NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
    SEQUENTIAL or FIXED-N PLAN (S or F):  F

LOWER, UPPER, OR BOTH     (L, U, or B):  L
                     LOWER SPEC LIMIT =  0
   IS SIGMA KNOWN (Y/N):  Y     SIGMA =  2.7    <---
               KIND OF INPUT  (P or D):  P
         ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10
D-RULE, P-CURVES, S-STATS(D, P, or S):  D


OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F     APPEND/WRITEOVER (A/W): A

                    DECIMAL PLACES =  2

       n = 10          LA= 4.870664
```

OUTPUT SCREEN

```
        *****   DESIGN JOURNEY:  PLAN #6   *****

   FIXED SAMPLE SIZE VARIABLES SAMPLING PLAN TO CONTROL
           FRACTION CONFORMING TO:  LISL = 0
   FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT   SIGMA = 2.7

   PRODUCERS POINT                        CONSUMERS POINT
      AQL = .01                               RQL = .10
    ALPHA = .05                              BETA = .05


    DECISION RULE FOR A FIXED-N SAMPLING PLAN:

        SAMPLE SIZE = 10
        REJECT IF XBAR IS LESS THAN 4.87

    BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994
```

STEP 6.1

PURPOSE: Design a variables plan, fixed-n, known-sigma.

This input screen shows how to design a known sigma sampling plan for variables.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) Known sigma plans are appropriate when the within-lot standard deviation is stable from lot to lot. This stability can be evaluated by statistical tests, including S-Charts and R-Charts.
2) The sample average is used to make the acceptance decision.
3) Good practice is to evaluate the standard deviation for each lot. If the standard deviation has changed, switch to a matched plan for unknown-sigma.

THE RESULT: Variables plan, n=10, A=4.87 known-sigma.

DESCRIPTION OF THE OUTPUT REPORT

The report contains the producer's and consumers' points, and the decision rule for acceptance or rejection of the lot.

INTERPRETATION OF THE OUTPUT REPORT

The plan: n=10, A=4.87 shows that known sigma plans provide substantial reduction in sample size.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, in step 7, we will design a range plan to check the assumption that the standard deviation has not changed.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)    Two-Point Method of Design      Sampling Plans for VARIABLES

           PRODUCT OR PROCESS NAME:  PROCESS XXX
                     VARIABLE NAME:  MEASUREM'T
FORM: ISL, MEAN, RANGE (I, M, or R):  R


              WITHIN-LOT SIGMA =  2.7


          TRUNCATION SAMPLE SIZE =  10

OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F     APPEND/WRITEOVER (A/W): W

                  DECIMAL PLACES =  2
ENTER A TITLE:
```

STEP 7.1

PURPOSE: Design a Range Plan to test the assumption of known sigma.

This input screen shows how to design a sampling plan for the range to test the assumption of known sigma.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) The fact that a process has had a stable "known" standard deviation in the past does not completely assure the current lot hasn't changed.
2) A variables sampling plan, whether fixed-n or sequential, should be accompanied by a range plan. This simple test checks whether the assumption of known sigma still holds for the current lot.
3) To reject with a range plan is to disqualify the known sigma plan. A common course of action is to switch to the equivalent sigma-unknown plan. This will usually lead to more samples.

PROGRAM OPERATION

1) Choose the range option, R.
2) Enter the "known" within-lot sigma of 2.7.
3) Enter a truncation sample size equal to the maximum n of the sequential or fixed-n plan.

OUTPUT SCREEN

```
        *****   DESIGN JOURNEY:   PLAN #7   *****

   RANGE RULE FOR THE ASSUMPTION OF KNOWN WITHIN-LOT SIGMA
               FOR PROCESS XXX MEASUREM'T
          TEST OF WITHIN-LOT SIGMA= 2.7 AT ALPHA=0.05

DECISION RULE:
  COMPARE THE SAMPLE RANGE TO THE DECISION LIMITS
  SAMPLE SIZE IS CONTROLLED BY THE PLAN FOR XBAR.

                  SAMPLE   REJECT IF RANGE
                   SIZE:   IS MORE THAN:
                   (N)         (Re)

                    2          7.48
                    3          8.94
                    4          9.80
                    5         10.42
                    6         10.88
                    7         11.26
                    8         11.58
                    9         11.85
                   10         12.07

     BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-22-1994
```

THE RESULT: Rejection table for the Range

DESCRIPTION OF THE OUTPUT REPORT

The table contains only rejection numbers, not Ac. The sample size is matched to the n=10 known-sigma plan of step 6. Sample size 2-9 of the range plan is for use with the matched sequential plan of step 8 to follow.

INTERPRETATION OF THE OUTPUT REPORT

If the sample range exceeds Re, conclude that the standard deviation of the lot is greater than the "known" sigma of 2.7.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, in step 8, we will design a known-sigma *sequential* sampling plan.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)    Two-Point Method of Design      Sampling Plans for VARIABLES

            PRODUCT OR PROCESS NAME:  PRODUCT XXX
                      VARIABLE NAME:  MEASUREM'T
       INPUT FORM, ISL or MEAN (I or M):  I
        NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
     SEQUENTIAL or FIXED-N PLAN (S or F):  S     <---
   CLASSICAL SEQUENTIAL or TSS  (C or T):  T
   LOWER, UPPER, OR BOTH     (L, U, or B):  L
                      LOWER SPEC LIMIT =  0
     IS SIGMA KNOWN (Y/N):  Y    SIGMA =  2.7   <---
              KIND OF INPUT  (P or D):  P
          ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10
   D-RULE, P-CURVES, S-STATS(D, P, or S):  D


OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F    APPEND/WRITEOVER (A/W): A


                   DECIMAL PLACES =  2

        n = 10          LA= 4.870664
```

STEP 8.1

PURPOSE: Design a sequential variables plan, known-sigma.

This input screen shows how to design a sequential variables plan with known standard deviation.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) A sequential sampling plan will reduce the sample size compared to a fixed-n plan.
2) Two kinds of sequential plans are available:
   - Classical Sequential Probability Ratio(SPR) plans
   - Truncatable Single Sample(TSS) plans

We will use a TSS plan here because the continue zone of TSS plans truncate naturally at the sample size and decision limit of the fixed-n plan. (n=10, A=4.87).

OUTPUT SCREEN

```
          *****   DESIGN JOURNEY:  PLAN #8   *****

     TSS SEQUENTIAL VARIABLES SAMPLING PLAN TO CONTROL
          FRACTION CONFORMING TO:  LISL = 0
  FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT    SIGMA = 2.7

  PRODUCERS POINT                          CONSUMERS POINT
     AQL = .01                                 RQL = .10
   ALPHA = .05                                BETA = .05

  DECISION RULE:
    COMPARE THE SAMPLE AVERAGE TO THE DECISION LIMITS
    CONTINUE SAMPLING UNTIL ACCEPTANCE OR REJECTION:
```

SAMPLE SIZE (N)	REJECT IF LESS THAN (Re)	ACCEPT IF MORE THAN (Ac)
1	-2.06	11.80
2	0.31	9.43
3	1.45	8.29
4	2.18	7.56
5	2.72	7.02
6	3.17	6.57
7	3.55	6.19
8	3.92	5.83
9	4.28	5.46
10	4.87	4.87

```
  BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994
```

THE RESULT: Sequential decision rule, known-sigma.

DESCRIPTION OF THE OUTPUT REPORT

The report contains the values of:

- The specification: LISL=0
- The known standard deviation: sigma=2.7
- The producer's and consumers' points: (AQL,alpha) & (RQL,beta)
- The decision rule instructions
- A table of decision limits vs sample size

INTERPRETATION OF THE OUTPUT REPORT

The decision table shows that very high and very low sample averages will lead to an accept/reject decision with small sample size.

THE NEXT STEP OF THE DESIGN JOURNEY

Next we will evaluate the performance of this known-sigma variables plan with OC, AOQ, ASN, and ARL curves.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)    Two-Point Method of Design     Sampling Plans for VARIABLES

           PRODUCT OR PROCESS NAME:  PRODUCT XXX
                     VARIABLE NAME:  MEASUREM'T
     INPUT FORM, ISL or MEAN (I or M):  I
      NAME OF THE NONCONFORMING UNIT:  BEYOND LIMIT
   SEQUENTIAL or FIXED-N PLAN (S or F):  S
CLASSICAL SEQUENTIAL or TSS  (C or T):  T
LOWER, UPPER, OR BOTH     (L, U, or B):  L
                  LOWER SPEC LIMIT =  0
   IS SIGMA KNOWN (Y/N):  Y    SIGMA =  2.7
            KIND OF INPUT  (P or D):  P
       ENTER ALPHA, BETA, AQL, RQL =  .05 .05 .01 .10
D-RULE, P-CURVES, S-STATS(D, P, or S):  P    <---
OUTPUT FORMAT: GRAPH OR TABLE(G or T):  T    <---
 COLUMNS IN TABLE: (O, P, M, N, Q, L):  ALL  <---
      STEPPING VARIABLE (O, P, or M):  M    <---
OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  F     APPEND/WRITEOVER (A/W): A

  SCALE LIMITS FOR THE MEAN: MIN,MAX =  4 14 <---
                    DECIMAL PLACES =  2

       n = 10          LA= 4.870664
```

STEP 8.2

PURPOSE: Analyze known-sigma plans with a performance table.

This input screen shows how to produce a table of all possible performance curves for the sequential variables plan that matches the fixed-n plan: n=10, k=1.80.

SAMPLING PLAN DESIGN CONSIDERATIONS

We obtain a table of all performance curves. The curves are defined in step 3.3. For the stepping variable, we selected the lot mean.

INTERPRETATION OF THE OUTPUT REPORT

1) The first two columns show how the fraction defective, p', increases as the mean decreases for this lower limit plan.
2) Columns (1) vs (3) form the OC-Curve based on the lot mean.
3) Columns (2) vs (3) form the OC-Curve based on the fraction defective.
4) The ASN column shows that:
   - ASN maximum is 7.9.
   - ASN=4.9 at mean= 6.25 and p'=0.010 (AQL)
   - ASN=2.9 at mean= 7.50 and p'=0.003
   - ASN=1.0 at mean=12.25 and p'<0.000005
5) This information allows you to set process targets in an informed way to reach the desired ASN.

OUTPUT SCREEN

```
***** DESIGN JOURNEY: PLAN #8 *****

TABLE OF PERFORMANCE CHARACTERISTIC CURVES
OF VARIABLES SAMPLING PLAN TO CONTROL
FRACTION CONFORMING TO: LISL = 0
FOR PRODUCT XXX MEASUREM'T BEYOND LIMIT   SIGMA = 2.7

PRODUCERS POINT                 CONSUMERS POINT
  AQL = .01                        RQL = .10
  ALPHA = .05                      BETA = .05
```

TRUE MEAN MEASUREM'T (MEAN)	TRUE FRACTION BEYOND LIMIT (p')	PROBABILITY OF ACCEPTANCE (Pa)	AVERAGE SAMPLE NUMBER (ASN)	AVERAGE RUN LENGTH (ARL)	AVERAGE OUTGOING QUALITY (AOQ)
14.00	0.00000	1.0000	1.0		0.00000
13.75	0.00000	1.0000	1.0		0.00000
13.50	0.00000	1.0000	1.0		0.00000
13.25	0.00000	1.0000	1.0		0.00000
13.00	0.00000	1.0000	1.0		0.00000
12.75	0.00000	1.0000	1.0		0.00000
12.50	0.00000	1.0000	1.0	8380000	0.00000
12.25 ***	0.00000	1.0000	1.0 ***	5590000	0.00000
12.00	0.00000	1.0000	1.1	2790000	0.00000
11.75	0.00001	1.0000	1.1	1670000	0.00001
11.50	0.00001	1.0000	1.1	1040000	0.00001
11.25	0.00002	1.0000	1.2	599000	0.00002
11.00	0.00002	1.0000	1.2	356000	0.00002
10.75	0.00003	1.0000	1.3	215000	0.00003
10.50	0.00005	1.0000	1.4	127000	0.00005
10.25	0.00007	1.0000	1.4	75200	0.00007
10.00	0.00011	1.0000	1.5	44700	0.00011
9.75	0.00015	1.0000	1.6	26500	0.00015
9.50	0.00022	0.9999	1.6	15700	0.00022
9.25	0.00031	0.9999	1.7	9340	0.00031
9.00	0.00043	0.9998	1.8	5540	0.00043
8.75	0.00060	0.9997	2.0	3280	0.00060
8.50	0.00082	0.9995	2.1	1950	0.00082
8.25	0.00112	0.9991	2.2	1150	0.00112
8.00	0.00152	0.9985	2.4	688	0.00152
7.75	0.00205	0.9976	2.6	408	0.00205
7.50 **	0.00274	0.9959	2.9 **	242	0.00273
7.25	0.00362	0.9931	3.2	144	0.00360
7.00	0.00476	0.9884	3.5	86.2	0.00471
6.75	0.00621	0.9806	3.9	51.5	0.00609
6.50	0.00803	0.9677	4.4	31.0	0.00777
6.25 *	0.01031	0.9468	4.9 *	18.8	0.00976
6.00	0.01313	0.9135	5.6	11.5	0.01200
5.75	0.01660	0.8624	6.3	7.26	0.01432
5.50	0.02082	0.7881	7.0	4.72	0.01641
5.25	0.02592	0.6882	7.6	3.20	0.01784
5.00	0.03202	0.5671	7.9	2.30	0.01816 *
4.75	0.03927	0.4374	7.9	1.77	0.01717
4.50	0.04779	0.3157	7.6	1.46	0.01509
4.25	0.05774	0.2149	7.0	1.27	0.01241
4.00	0.06924	0.1397	6.3	1.16	0.00967

BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994

THE RESULT: OC, ASN, ARL, AOQ Curves with xbar and p'.

CONCLUSIONS:

1) This plan meets the sampling requirement specified on page 1 -- that lot with .10 fraction defective should not be released.
2) This analysis provides a basis for achieving an economic balance between:
   - The cost of inspection.
   - The cost of targeting the mean.
   - The consequence of shipping defective items.

THE NEXT STEP OF THE DESIGN JOURNEY

Next, in step 9, we will design a plan to control the process mean to a target.

DESIGN JOURNEY

An Example of Applying Two-Point Sampling Plans

INPUT SCREEN

```
TP414 (v1.21)     Two-Point Method of Design      Sampling Plans for VARIABLES

           PRODUCT OR PROCESS NAME:  PROCESS XXX
                     VARIABLE NAME:  MEASUREM'T
FORM: ISL, MEAN, RANGE (I, M, or R):  M   <---

SEQUENTIAL or FIXED-N PLAN (S or F):  F   <---

LOWER, UPPER, OR BOTH    (L, U, or B):  B   <---

                            SIGMA =  2.7
          KIND OF INPUT  (P or D):  P
ALPHA, BETA, LRQL, LAQL, UAQL, URQL =  .00135 .5 3.45 7.5 7.5 11.55   <---
   KIND OF OUTPUT REPORT(D, P, or S):  D


OUTPUT TO SCREEN,PRINTER,FILE (S,P,F):  P


                   DECIMAL PLACES =  2

     n = 4        LA= 3.450015          UA= 11.54998
```

OUTPUT SCREEN

```
          *****   DESIGN JOURNEY:   PLAN #9   *****


   FIXED SAMPLE SIZE VARIABLES SAMPLING PLAN TO CONTROL
        THE MEAN WITH BOTH LOWER AND UPPER LIMITS
       FOR PROCESS XXX MEASUREM'T    SIGMA = 2.7

PRODUCERS POINT                          CONSUMERS POINT
AQLML =  7.50                              RQLML =  3.45
AQLMU =  7.50                              RQLMU = 11.55
ALPHA = .00135                              BETA = .50000


     DECISION RULE FOR A FIXED-N SAMPLING PLAN:

          SAMPLE SIZE = 4
          REJECT IF XBAR IS LESS THAN 3.45
          REJECT IF XBAR IS MORE THAN 11.55

     BY SAMPLING PLAN PROGRAM TP414 V1.21 ON 03-26-1994
```

STEP 9.1

PURPOSE: Design a targeted sampling plan for the mean.

This input screen shows how to design a targeted sampling plan for the mean of a process.

SAMPLING PLAN DESIGN CONSIDERATIONS

1) We can assure high quality and low sample size by targeting the mean far above the lower specification limit (LISL=0). We will target the mean to 7.50, which is 2.8 standard deviations above LISL. We used the performance table in step 8.2 choose this process target. It shows that for a mean=7.50: p'=0.00274, Pa=.9959, ASN=2.9, ARL=242
2) Mean plans require that AQL and RQL be expressed in units of the mean of the quality characteristic, rather than fraction nonconforming. We use the symbols M for mean, L for lower, and U for upper.
3) We followed these steps in designing the targeted sampling plan for the process mean:
   a) Let AQLML=AQLMU=Target=7.50.
   b) Let lower and upper RQLs be RQLML=3.45 RQLMU=11.55.
   c) We will use a 3*sigma control chart approach, which is two sided. For alpha=.0027, enter alpha/2 =.00135.
   d) This example will use Beta=0.50 to match the control chart concept. This will match an xbar chart where RQLMU=UCL, RQLML=LCL.

THE RESULT: Fixed-n decision rule for xbar.

DESCRIPTION OF THE OUTPUT REPORT

The decision rule is: Accept if xbar for n=4 is between 3.45 and 11.55 (inclusive)

INTERPRETATION OF THE OUTPUT REPORT

Note that alpha=.00135 is really alpha/2. The overall alpha risk for the targeted plan is .0027.

STRATEGY FOR USE

If the process mean is on target at 7.50, step 8.2 showed ASN=2.9 for the sequential product plan. Therefore the sequential plan would almost always make a decision by n=4, and no further measurements will be required.

This xbar plan can be implemented as an xbar control chart to make use of the SPC methods for identifying between-lot sources of variability in the process.

To draw an xbar control chart, use LCL=3.45, center line=7.50, UCL=11.55, and a subgroup sample size of n=4.

RETRACING THE STEPS OF DESIGN JOURNEY

We started with a product acceptance plan that did not protect the customer. We followed these steps:

1) Attributes: n=3, C=0
2) Attributes: n=43, C=1
3) Attributes: Sequential Sampling Plan
4) Variables: Unknown-Sigma
5) Variables: Unknown-Sigma, Sequential
6) Variables: Sigma-Known
7) Variables: Range Plan
8) Variables: Sigma-Known, Sequential Plan
9) Targeted Xbar control chart for the process mean

Two-Point Product Acceptance Sampling Plans

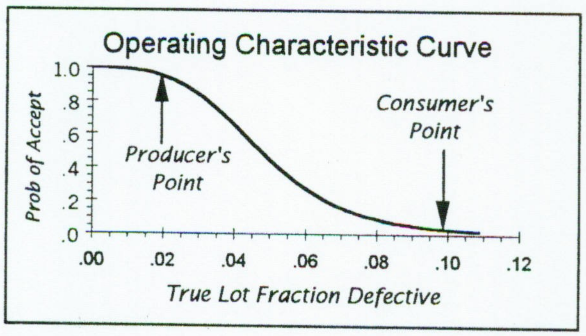
The Two-Point approach has important advantages over Mil-Std 105 and Mil-Std 414

TWO-POINT SAMPLING PLANS

The Two-Point[1] method develops sampling plans for product acceptance. You specify two points of the **operating characteristic curve** (oc-curve).

- The **producer's point** controls the probability of accepting lots that are at an acceptable quality level. (See figure)
- The **consumer's point** controls the risk of accepting lots that are at a rejectable quality level. (See figure)

The **oc-curve** enables you to evaluate the acceptance probability for any true lot quality level -- on a what-if basis. This way, you can design sampling plans that perform the way you want.



MIL-STD PLANS

The Mil-Standard plans require you to choose the producers point only -- not the consumer's point. Actually, the consumer's point might be more important. A producer expects an acceptance plan **not to accept poor quality lots**.

STRAIGHTFORWARD TO USE

Two-Point sampling plans involve only the performance of the decision rule, as described by two points on the oc-curve. You do not have to use the esoteric 'inspection levels' and 'code letters' required by the Mil-Standards.

LOT-SIZE

The Two-Point approach does not use lot size to determine the oc-curve, whereas the Mil-Standards do[2]. The lot size does not effect the sampling risks when sampling large lots with small samples -- lot size > 10*(sample size).

SEQUENTIAL ACCEPTANCE

The Two-Point software programs provide the option of sequential sampling. These plans allow the operator to increase the sample size one item at a time, or to form it into groups to match the logistics of the situation. This is the most efficient type of plan.

Mil-Std 105 contains double and multiple plans comparable to sequential. Mil-Std 414 does not have a comparable scheme.

THE FALLACY OF "AQL ASSURANCE"

A widespread **misconception** states that a sampling plan can ensure that the quality of accepted lots will not exceed AQL. The two-point approach avoids this with probabilities that show that any plan **can accept** quality worse than AQL -- **sometimes much worse.**

Thus Two-Point sampling plans **prevent** this important misconception that often accompanies the use of the Mil-Standards.

NONINTUITIVE IMPROVEMENTS

The two-point approach can reveal relationships that are not intuitive. For example, we can reduce the sample size substantially by changing from:
(AQL=.05, RQL=.10) to
(AQL=.01, RQL=.05).

Here, better quality levels require less sampling, when done correctly.

PARTS PER MILLION

The two-point software will design plans that detect quality levels in the part-per-million range. The Mil-Standards limit you to using tabulated levels.

SUPPORT OF MIL-STDS

The need for team agreement might require use of Mil-Stds 105 or 414 sampling plans. For example, your customer might have commitment to Mil-Std 105, or your vendor might have previously invested in Mil-Std training for inspectors and operators.

In these instances, the Two-Point approach provides a rational basis for designing specific sampling plans. Then determine the required Mil-Std inspection levels, etc., to have the same oc-curve.

KNOWLEDGE OF PERFORMANCE

From the consumer, regulatory, and litigation standpoints, it is safer to know the two probabilities of a decision rule than simply to be able to say that you meet a published standard.

---

[1] The acronym **TP** stands for Two-Point in the software program names **TP**105, **TP**414, and **TP**781.

[2] Mil-Stds 105 and 414 use lot size to determine specific sampling plans, but they assume infinite population to calculate oc-curves.

Matched Decision Rules

All of these decision rules match ***the same*** *Operating Characteristic Curve*

#2: ATTRIBUTE FIXED-n: TP105

n=43 C=1

#4: VARIABLES FIXED-n: TP414
UNKNOWN SIGMA

n=27 K=1.80

#6: VARIABLES FIXED-n: TP414
KNOWN SIGMA = 2.7

n=10 A=4.94

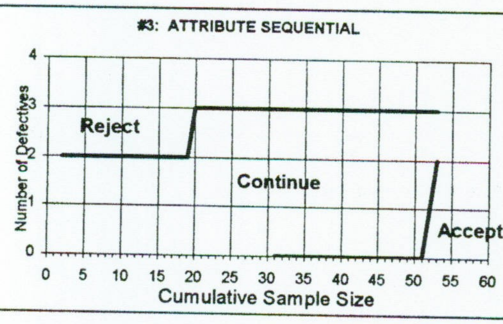
Protection for the Producer and Consumer

The exceptional adaptability of the Two-Point approach cannot be equaled by any other method. These seven sampling plans have identical ability to discriminate between good and bad quality. They all have identical producer's and consumer's points (AQL=.01, Alpha=.05 RQL=.10, Beta=.05). The difference is in efficiency, cost, and ease of application. The decision tables shown below were generated by TP105 and TP414.The sequential graphs were drawn by Microsoft Excel (not included)..

NOTE: The numbering system - #2 to #7 - matches Design Journey. Plans #6 & #7 use ISL=.07, so they differ from Design Journey.

TP105:

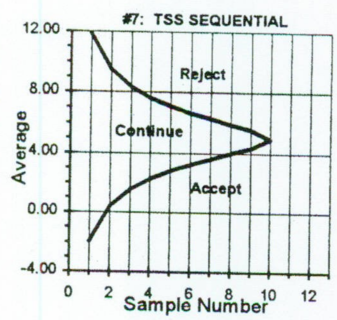
From	To	(Ac)	(Re)
1	1	*	**
2	19	*	2
20	30	*	3
31	51	0	3
52	52	1	3
53	53	2	3



#7 KNOWN SIGMA = 2.7

TP414:

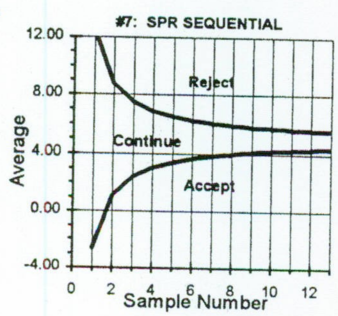
(n)	(Ac)	(Re)
1	-1.99	11.87
2	0.38	9.50
3	1.52	8.36
4	2.25	7.63
5	2.79	7.09
6	3.24	6.64
7	3.62	6.26
8	3.98	5.89
9	4.35	5.53
10	4.94	4.94



#7 KNOWN SIGMA = 2.7

TP414:

(n)	(Ac)	(Re)
1	-2.67	12.55
2	1.13	8.74
3	2.40	7.48
4	3.04	6.84
5	3.42	6.46
6	3.67	6.21
7	3.85	6.03
8	3.99	5.89
9	4.09	5.78
10	4.18	5.70
11	4.25	5.63
12	4.31	5.57
13	4.36	5.52



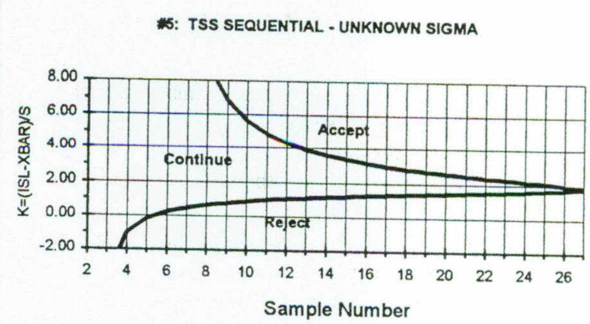
METHOD OF CHARTING

The sequential charts on this page were produced by exporting the data tables from the Two-Point sampling software to a file. The file was then imported to an Excel (not included) worksheet, where the charts were produced.

TP414:

#5 UNKNOWN SIGMA

(n)	(Re)	(Ac)
2		
3	-3.83	
4	-1.03	
5	-0.20	
6	0.21	
7	0.46	
8	0.63	8.94
9	0.76	6.86
10	0.87	5.66
11	0.96	4.88
12	1.03	4.33
13	1.10	3.92
14	1.16	3.61
15	1.21	3.36
16	1.26	3.15
17	1.30	2.98
18	1.35	2.83
19	1.39	2.69
20	1.43	2.58
21	1.47	2.47
22	1.51	2.37
23	1.55	2.27
24	1.59	2.18
25	1.64	2.08
26	1.70	1.96
27	1.80	1.80



FACTSHEET FOR DESIGN OF TWO-POINT SAMPLING PLANS

Software Programs: TP105, TP414, and TP781

TWO-POINT

TP105, TP414, and TP781 are user-friendly programs that develop sampling plans by the two-point method of design – where the user specifies and controls the consumer's point (RQL and consumer's risk) and the producer's point (AQL and producer's risk).

PERFORMANCE

A clear picture of the performance of the designed sampling plans is provided by OC, AOQ, ASN, and ARL curves. A user's manual explains these curves. Graphs are produced for OC-curves and sequential decision rules.

FIXED-N/SEQUENTIAL

All three programs design fixed-n and sequential sampling plans. They can also evaluate existing fixed-n plans for adequacy, and can convert fixed-n plans into sequentials.

FLEXIBILITY

Output can be routed to screen, printer, or a file. The ASCII file can be read by other software:

1. by many spreadsheet and graphics programs for plotting and presentations
2. by many word processors for inclusion in documents, standards, and procedures.

TABLE OF PROGRAM FEATURES AND SPECIFICATIONS

Program	Kind of Plan Designed	Quality Levels {AQL & RQL UNIT}	Probability Distribution	Performance Curves
TP105 Version 1.2 $245.00*	Attribute	Fraction Defective	Binomial	OC, ASN, AOQ, ARL
	Count	Mean Count	Poisson	OC, ASN, AOQ, ARL
TP414 Version 1.2 $245.00*	Mean	Mean	Normal, Known Sigma	OC, ASN, ARL
	ISL { INDIVIDUAL SPECIFICATION LIMIT }	Fraction Nonconforming	Normal, Known Sigma, Unknown Sigma	OC, ASN, AOQ, ARL Mean -vs- p'
TP781 Version 1.2 $245.00*	MTBF	Mean Time Between Failures	Exponential	OC, ASN
	Mission	Probability of Mission Success	Exponential	OC, ASN

* **DISCOUNT:** Deduct for multiple programs: $50 off total for two programs, $125 off total for three programs.

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HOW TO DESIGN A TWO-POINT SAMPLING PLAN

The Two-Point method aims at minimizing the two wrong decisions that you can make when sampling: (1) to reject a good lot and (2) to accept a bad lot.

The most effective way to improve your accept/reject decisions is by using the Two-Point method of sampling plan design. This article explains important visualization tools of this method: (1) decision tables and (2) OC-Curves.

USE DECISION TABLE TO DEFINE PLAN

The decision table explains Type I and Type II errors. Use it to define your sampling requirement. *See figure 1*

QUALITY OF LOT	RESULT OF DECISION: ACCEPT the Lot	RESULT OF DECISION: REJECT the Lot
Good Lot (AQL)		Producer's Worry TYPE I ERROR Producer's Risk = Alpha
Bad Lot (RQL)	Consumer's Worry TYPE II ERROR Consumer's Risk = Beta	

Figure 1. Decision Table

A Type I error is a false rejection decision:
It is to reject a good lot, i.e., to reject when the true value of the quality characteristic[1] is AQL[2]. The risk of rejecting a good lot is the producer's risk (Alpha risk, α).

A Type II error is a false acceptance decision:
It is to accept a bad lot, i.e., to accept when the true value of the quality characteristic is RQL[3]. The risk of accepting a bad lot is the consumer's risk (Beta risk, β).

The Two-Point Method described:
The Two-Point method describes the accept/reject decision goal by specifying the two errors. Do this by choosing alpha, beta, AQL, and RQL. Then calculate the sample size (n) and the decision limit(s).

The Two-Point software programs TP105, TP414, and TP781 inputs alpha, beta, AQL, and RQL to calculate the sample size and decision limit(s).

USE OC-CURVE TO EVALUATE PLAN

The OC (Operating Characteristic) Curve is a tool which also addresses the Type I and Type II error, but in a slightly different way from the decision table: The Type I and Type II errors define two points on the OC-Curve.

The OC-Curve shows the probability of acceptance, Pa, for any level of lot quality. *Figure 2.* On the horizontal axis is the quality characteristic. Interpret the curve according to this example: "If the quality is 0.09, then Pa is 0.05."

Two special points on the OC-Curve are the producer's point (AQL,1-Alpha) and consumer's point (RQL,Beta).

i.e. The producer made a good lot but it is rejected, or the consumer receives a bad lot because it is accepted.

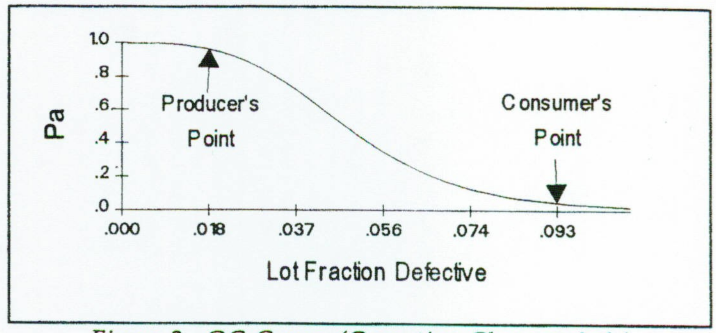

Figure 2. OC-Curve. (Operating Characteristic)

How to choose the Producer's Point in practice
You should expect that lots at the producer's point (AQL) will be accepted most of the time. Define AQL accordingly. Relate to known historical quality levels. Also specify the producers risk of rejecting a lot that is of AQL quality. Typical: alpha = 0.05. (5%)

How to choose the Consumer's Point in practice
You should expect that lots at the consumer's point quality level (RQL) will be rejected most of the time. Define RQL accordingly. Also specify the consumers risk of accepting a lot that is RQL. Typical: beta = 0.05.

The Two-Point software programs TP105, TP414, and TP781 can produce the OC-Curve for any alpha, beta, AQL, and RQL. They also produce OC-Curves from input of decision rules (n & C, etc.)

USE OC-CURVES TO COMPARE PLANS

OC-Curves compare alternative plans. *Figure 3.* Choose between the plans by their relative performance. You should expect that the steeper the curve, the larger the sample size.

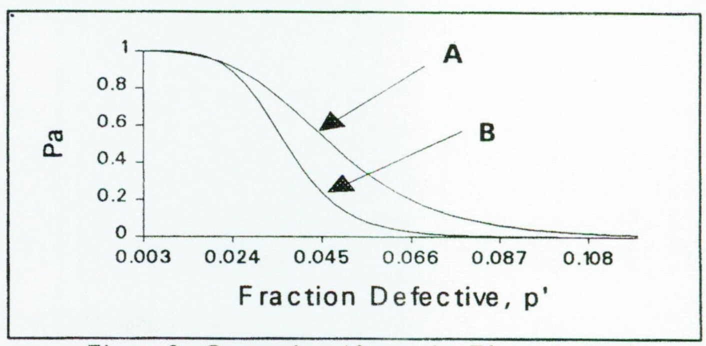

Figure 3. Comparing Alternative Plans, A and B

Complete this picture by comparing the costs of the sampling to the resulting performance.

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1 The quality characteristic can be any quality statistic : fraction defective, defects per unit, the mean, the standard deviation, Mean Time Between Failures, etc.

2 AQL is the Acceptable Quality Level. AQL is also called PQL for Producer's Quality Level. It is good quality.

3 RQL is the Rejectable Quality Level. RQL is also called LTPD, LQ, and CQL. It is bad quality.